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ABSTRACT

The purpose of this study was to assess the dimensionality of attitudinal data arising from unfolding models for discrete data and to compute rough estimates of item and individual parameters for use as starting values in other estimation parameters. One- and two-dimensional simulated test data were analyzed in this study. Results of limited analyses performed so far have shown that linear principal components analysis of unfolding data provides a reliable estimate of the underlying dimensionality. For every unfolding dimension, there are two linear principal components. In addition, pattern coefficients of items on the two principal components associated with each dimension form a fan-shaped simplex pattern resembling a semicircle. Arc length of an item along the semicircle can serve as an estimate of item location. Length of an item in the item space spanned by the two linear components can serve as an estimate of item discrimination. For individuals, arc lengths computed using the individual scores on the two principal components could serve as estimates of individuals' location parameters. For two dimensional test data, an algorithm has been developed to identify component pairs associated with each dimension and to classify items correctly into dimensional groups. (Contains 1 table, 7 figures, and 16 references.) (Author/SLD)

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Attitudinal Data: Dimensionality and Start Values for Estimating Item Parameters

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Abstract

The purpose of this study was to assess dimensionality of attitudinal data arising from unfolding models for discrete data, and to compute rough estimates of item and individual parameters for use as starting values in other estimation programs. One- and two-dimensional simulated test data were analyzed in this study. Results of limited analyses performed so far have shown that linear principal components analysis of unfolding data provides a reliable estimate of the underlying dimensionality. For every unfolding dimension, there are 2 linear principal components. In addition, pattern coefficients of items on the two principal components associated with each dimension form a fan-shaped simplex pattern resembling a semicircle. Arc length of an item along the semicircle can serve as an estimate of item location. Length of an item in the item space spanned by the two linear components can serve as an estimate of item discrimination. For individuals, arc lengths computed using the individual scores on the two principal components could serve as estimates of individuals' location parameters. For two dimensional test data, an algorithm has been developed to identify component pairs associated with each dimension and to correctly classify items into dimensional groups.

Attitudinal Data: Dimensionality and Start Values for Estimating Item Parameters

Increasingly, attitudinal data are being used along with achievement data to assess educational outcomes. For example, it is important to measure teacher and pupil attitude or opinion about mathematics when assessing the effectiveness of mathematics curriculum in elementary grades. This is typically accomplished by having respondents indicate the degree to which they agree (or disagree) to a series of attitude statements.

In the framework of unfolding theory it is presumed that a subject endorses an item (statement) if the content of the item matches the subject's opinion. Psychometrically speaking, we would expect more endorsement to the extent that a subject is located close to an item on a latent attitudinal space. Let δ_i denote the item location on the latent continuum and θ_j denote the subject's location on the same latent continuum. Then, the degree of agreement with the statement increases as $|\theta_j - \delta_i|$ approaches zero. And the degree of disagreement with the item increases as this difference $|\theta_j - \delta_i|$ increases, resulting in a non-monotone, bell shaped, single-peaked response function for the given item.

Several stochastic models have been recently developed to model attitudinal data arising from an unfolding model. Parametric item response models include the squared simple logistic model (Andrich, 1988), the PARELLA model (Hoijtink, 1990, 1991), hyperbolic cosine model (Andrich & Luo, 1993; Luo, 2001), the graded unfolding model (Roberts & Laughlin, 1996), and the generalized graded unfolding model (Roberts, Donoghue, & Laughlin, 2000). Nonparametric item response models also have been proposed for data resulting from an ideal point response process (Cliff, Collins, Zatkin, Gallipeat, & McCormick, 1988; van Schuur, 1984).

Although there are several models available to analyze attitude data resulting from an ideal point process, there are no well-established procedures to assess model data fit, which is fundamental for proper interpretation of resulting estimates. If a unidimensional model is applied to analyze data, for example, it is important to establish a unidimensional trait underlying the data before applying the model to estimate an individual's scale value. Even though many methodologies have been developed for dimensionality assessment of achievement item data, they cannot be readily applied to attitude item data because models for attitudinal data implement

single peaked, non-monotone response functions, while models for achievement data follow cumulative monotone response functions.

Davison (1977) showed that when item responses follow a simple metric unfolding model, the principal components of the inter-item correlation matrix will suggest two primary components, and the loadings of the components form a simplex pattern. Davison used a simple unidimensional model to demonstrate his results, namely, the squared distance model, where item responses are linearly related to continuous squared distances between item and person scores. A paper by van Schuur and Kiers (1994) has provided a comprehensive summary of relationship between unfolding models and linear factor analysis. They showed that when unidimensional data arising from an unfolding model are analyzed using traditional linear factor analysis, one obtains two independent factors instead of one bipolar factor. This is because, in linear factor analysis, observed variables are linearly related to the underlying latent variables or dimensions. Whereas in unfoldable data, observed variables have a non-linear relationship to underlying dimensions, and also the relationship is non-monotonic. Ross and Cliff (1964) mathematically proved that if the underlying dimensionality of unfolding data is r, then principal components analysis of such data will give a factor solution of dimension $r+1$, using a squared distance model. Maraun and Rossi (2001) have demonstrated that for unfoldable data, the unidimensional model is equivalent to the unidimensional quadratic factor model, and that items conforming to r-dimensional space will have $2r$-dimensional representation using linear factor analyses.

All these studies have used simple, continuous distance models for simulating unfolding data, and for the most part, have employed only principal components analysis with unities in the diagonal for dimensionality assessment. More importantly, these studies were highly limited in their empirical investigation of dimensional structure of data.

The objectives of the present study were two-fold: (1) to conduct a systematic empirical investigation of dimensional structure of unfolding data using linear factor analysis when data are modeled via a discrete unfolding IRT model; (2) to establish start values of item and individual parameters for use in estimation programs for one and two-dimensional data.

This study is not yet fully completed. We did a mini dimensionality study to get a sense of the effect of different variables and types of factor analysis solutions to assess dimensionality of unfolding data. Since the results were promising, we did another mini study on parameter

estimation for one-dimensional unfolding data. We have just begun analysis of two-dimensional data. A full scale simulation study encompassing both dimensionality and parameter estimation will be undertaken shortly.

The Dimensionality Study

Given the limited empirical investigation of different approaches to determine the dimensional structure of unfolding data, the first study investigated the relationship between the true dimensionality of unfolding data and the number of linear factors generated for one- and two-dimensional tests.

The unidimensional simulation results presented in this study are based on the generalized graded unfolding model (GGUM) due to Roberts, Donoghue, and Laughlin (2000) as given by:

$$P[Z_i = z \mid \theta_j] = \frac{\exp\left(\alpha_i[z(\theta_j - \delta_i) - \sum_{k=0}^{z}\tau_{ik}\right) + \exp\left(\alpha_i[(M-z)(\theta_j - \delta_i) - \sum_{k=0}^{z}\tau_{ik}\right)}{\sum_{w=0}^{C}\left[\exp\left(\alpha_i[w(\theta_j - \delta_i) - \sum_{k=0}^{w}\tau_{ik}\right) + \exp\left(\alpha_i[(M-w)(\theta_j - \delta_i) - \sum_{k=0}^{w}\tau_{ik}\right)\right]} \quad (1)$$

where Z_i is the observable response to attitude statement *i*, taking values 0,1,2,…, C. A response value of C denotes strongest level of agreement and 0 denotes strongest level of disagreement, and $M = 2C+1$. Given a subject's attitude position on the latent continuum (θ_j), Equation 1 describes the probability that the response for person *j* to item *i* falls into a particular category as a function of the signed distance ($\theta_j - \delta_i$) between the person's attitude position θ_j and the location of the item δ_i. α_i is the discrimination parameter of the item, and τ_{ik} (k = 0,1,2,…C) are the threshold parameters which are symmetric about the point ($\theta_j - \delta_i$) = 0.

Individual attitude positions (θ_j) for unidimensional tests were generated from the standard normal distribution; Item discrimination parameters (α_i) were generated from a uniform distribution with parameters between .5 and 2. Item locations (δ_i) were generated from a uniform distribution between -2 and 2. The threshold parameter τ_{iC} was generated from a uniform distribution with parameters between -1.4 and -.4., and the successive thresholds were generated recursively as:

$$\tau_{ik-1} = \tau_{ik} - .25 + e_{ik-1}, \text{ for } k = 2,3,...,C, \text{ where } e_{ik-1} \sim N(0,04)$$

The two-dimensional results are based on a two-dimensional GGUM model, an extension of Equation 1 as given by:

$$P[Z_i = z \mid (\theta_{1j}, \theta_{2j})] = \frac{\exp\left[\sum_{d=1}^{2}\left(\alpha_{di}[z(\theta_{dj} - \delta_{di}) - \sum_{k=0}^{z}\tau_{dik}\right)\right] + \exp\left[\sum_{d=1}^{2}\left(\alpha_{di}[(M-z)(\theta_{dj} - \delta_{di}) - \sum_{k=0}^{z}\tau_{dik}\right)\right]}{\sum_{w=0}^{C}\left[\exp\left[\sum_{d=1}^{2}\left(\alpha_{di}[w(\theta_{dj} - \delta_{di}) - \sum_{k=0}^{z}\tau_{dik}\right)\right] + \exp\left[\sum_{d=1}^{2}\left(\alpha_{di}[(M-w)(\theta_{dj} - \delta_{di}) - \sum_{k=0}^{z}\tau_{dik}\right)\right]\right]}$$

Individual attitude positions (θ_{1j} and θ_{2j}) for two-dimensional tests were generated from a bivariate normal distribution with appropriate correlation for two dimensions. Two levels of correlation between attitudinal dimensions were considered: 0 and 0.5. Location and threshold parameters were independently generated for the two dimensions as explained before. In generating item discrimination parameters α_{1i} and α_{2i}, two constants ξ_1 and ξ_2 were used to indicate the influence of each dimension on the item. For example, if $\xi_1 = \xi_2 = 0.5$, then both attitudes have equal weight on the item. On the other hand, if $\xi_1 = .75$ and $\xi_2 = .25$, then the first attitude influences the item more heavily than the second attitude. Item discrimination parameters were generated as follows. Initially, α_1 and α_2 were each generated from a uniform distribution between 0 and 1. Then they were appropriately linearly transformed using ξ_1 and ξ_2, so that final values of α_1 and α_2, each range between .5 and 2.

Two-dimensional test items were of two types: (1) simple structure, where each item had nonzero loading on only one attitude continuum; and (2) mixed structure, where some test items had non-zero loadings on both attitude continuums (complex items).

A Six-point scale was used ("strongly disagree", "disagree", "slightly disagree", " slightly agree", "agree", and "strongly agree") to score individual responses. For one-dimensional tests, the probability of each categorical response for an individual on an item was computed using the GGUM model given by Equation 1. For two-dimensional tests, the probability of each response category for an item was computed using Equation 2. These probabilities were used to divide a probability interval into six mutually exclusive and exhaustive segments, where each segment corresponds to a particular observable response category. A random number from a uniform

distribution was then generated and the probability segment containing the generated number was the response category for the individual. This process is referred to as the observed data.

In order to investigate the effect of random error in the data on dimensionality assessment, both error (observed) and error free data were considered. Error free data was generated using the expected values instead of probabilities to generate individual responses. This was done by taking the expected value of responses for each individual as follows:

$$E_i = 0 * P(0) + 1 * P(1) + 2 * P(2) + 3 * P(3) + 4 * P(4) + 5 * P(5),$$

where P(x) denotes the probability of obtaining a score x for a given attitude level obtained using Equation 1.

Both principal components analysis and factor analysis solutions were investigated for determining dimensionality of given data. The sample size was fixed to 2000 subjects. Two test lengths were used: 20 and 80 items.

Linear factor analysis was performed on observed data and on expected values (i.e., error-free data). Principal components analysis and principal axis factor analysis were each performed. In each case, the underlying dimensionality and the number of factors was determined based on the eigenvalues of a bootstrapped parallel analysis as described below (Buja & Eyuboglu, 1992).

Bootstrapped Parallel analyses -- A random data set was generated with the same number of subjects and items as the given data set. This was done by independently sampling responses to each item with replacement from the given data set. Principal components analysis and factor analysis were performed and eigenvalues were averaged over the repeated bootstrap samples. Average eigenvalues of random data over 10 repeated samples were plotted against the eigenvalues of observed (or expected) data. Dimensionality was determined as the number of eigenvalues above the intersection point of these two plots.

For given test data, the underlying latent dimensionality was determined based on four procedures: principal components analysis of observed data (PCOB), principal components of expected values (PCEX), factor analysis of observed data (FAOB), and factor analysis of expected values (FAEX). In all these procedures, the bootstrapped parallel analysis criterion was used to determine the number of underlying dimensions.

Results of the dimensionality study

Results of dimensionality analyses of unidimensional and two-dimensional data are displayed in Table 1. The first and second columns denote true dimensionality and the test size. The third column denotes the number of items loading on each dimension. For example, '35-35-10' denotes that 35 items load on attitude 1(θ_1), 35 items load on attitude 2 (θ_2), and 10 items both attitudes θ_1 and θ_2. The fourth column denotes weight given to each attitude. For example, '0.75, 0.25' denotes that θ_1 has a weight of .75 while θ_2 has a lower weight of .25. In other words, attitude 1 influences the item to a higher degree than attitude 2. The fifth column denotes the correlation between the attitude dimensions. The next four columns denote the estimated dimensionality of given test data for all four procedures (PCEX, PCOB, FAEX, FAOB) based on the bootstrapped parallel analysis criterion to determine dimensionality.

From Table 1 it can be seen that, in all cases, there is an over inflation of the number of underlying dimensions determined using factor analysis and principal components. However, there is a systematic and consistent pattern of over inflation of dimensionality for principal components analysis (PCOB and PCEX) but not so for factor analysis (FAOB and FAEX). Specifically, principal components analysis results generally showed that there are 2 linear components for each unfolding dimensionality. This is true also for tests containing mixed items and correlated attitudes. In addition, the number of estimated dimensions is the same for both observed and expected (error-free) data. Hence, the error in the data has not played a substantial role in determining the underlying dimensionality. However, for factor analysis, the number of estimated dimensions is unpredictable. The last column shows eigenvalues of principal components analysis of observed data for a randomly chosen trial. It can be seen that in one-dimensional cases the first large eigenvalue corresponds to the unfolding dimension; and in two-dimensional cases, the first two eigenvalues (much larger than the other two) correspond to the two unfolding dimensions.

Figures 1 through 7 show plots of factor loadings for principal components analysis of observed data. Figure 1 shows the plot for one-dimensional test with 20 items. It can be seen that pattern coefficients of the two linear components formed a clear simplex pattern. That is, plot of loadings formed a fan-like pattern resembling a semicircle. Figures 2 to 7 show plots for a two-dimensional test with 40 items. For a two-dimensional test there are four linear components resulting in six plots. Two of these plots, Figures 4 and 5, exhibited simplex pattern,

corresponding to the two latent unfolding dimensions. These figures indicate that components 1 and 4 go with dimension 1 and components 2 and 3 go with dimension 2. Examining such plots across all tests it was found that the simplex pattern between the associated components was unambiguously clear when the data exhibited a simple structure. However, as the simple structure was contaminated through adding of mixed items and/or the correlation between abilities, the simplexes had more scatter and resembled a unidimensional simplex pattern as the correlation approached .5.

In two-dimensional test data, plots of pattern coefficients were extremely useful in identifying the pairs of components associated with each unfolding factor. For example, sometimes components 1 and 4 were associated with the first dimension and components 2 and 3 with the second dimension. Other times components 1 and 3 were associated with the first dimension and components 2 and 4 with the second. By observing the plots one can identify the component pairs associated with each dimension, and items associated with the correct component pair.

Based on the results of this study, it is clear that principal components analysis of observed data with bootstrapped parallel analysis criterion to determine the number of underlying factors provided reliable evidence about the underlying dimensionality. There are 2 linear components associated with each unfolding dimension. Plots together with eigenvalues determine the dimensionality of data. Plots of pattern coefficients help to identify the factor pairs and structure in two-dimensional tests.

It appears that simplex pattern of pattern coefficients could be used to determine a rough scale of items on the underlying latent continuum. These values could serve as rational starting values for item parameters of the underlying model when iterative parameter estimation methods are used.

The Study to Determine Start Values for Estimation

Unidimensional Case (d=1)

In the case of unidimensional unfolding data, it has been shown that linear principal components analysis of such data give rise to two significant linear components and that the pattern coefficients of items corresponding to those components form a simplex pattern. That is,

pattern coefficients formed a fan-like pattern resembling a semicircle. It is proposed that items along the semicircle may be ordered to approximately determine their position on the latent continuum. To do this, the arc length is computed from a fixed point on the coordinate axis for each item, and these lengths are used as estimates of item locations on the latent continuum (i.e., δ values). These steps are listed in the Algorithm 1 explained below.

Similarly, respondents can also be ordered based on their scores on the two principal components forming the factor space. The arc length of each respondent is computed from a fixed point on the coordinate axis and respondents are ordered according to the arc length to determine their approximate location on the latent continuum. The algorithm for computing the arc length for items (or individuals) is described below. Respondents' positions computed in this manner can serve as estimates of their attitudes (θ_j). Item discrimination parameter (α_i) can also be estimated from the length of the item vector projected on the two corresponding components.

Algorithm 1: An algorithm to determine item (δ_i, α_i) and respondent (θ_j) parameters on the latent continuum.

For one-dimensional tests, all calculations are based on the first two principal components associated with the correlation matrix among the items. For items, (x, y) values are pattern coefficients, and for respondents they are component scores. Call the (x,y) space associated with these pattern coefficients, the item space, and call the (x, y) space defined by the component scores, the respondent space.

The estimate of the item discrimination parameter (α_i) is the length of the item vector in the item space. For δ_i and θ_j, the idea is to calculate arc lengths, in the item space for δ_i and in the respondent space for θ_j. Plots corresponding to the item space and the respondent space can be arranged so the points range mostly in the first ($x \geq 0, y \geq 0$) and fourth ($x \geq 0, y < 0$) quadrants (as in Figure 1).

Step 1: Fix a point on the unit circle. We use (1,0).

Step 2: Compute the projection length, *r* as

$$r_i = \sqrt{x_i^2 + y_i^2}$$

Step 3: Compute the arc length[1] of each item or subject from the fixed point (1,0).

$$\text{arc length} = \sin^{-1}\left(\frac{y_i}{r_i}\right) \quad \text{if } x \geq 0$$

$$= \text{sign(y)} \cdot \cos^{-1}\left(\frac{x_i}{r_i}\right) \quad \text{if } x < 0$$

where y is the pattern coefficient (item) or component score (respondent) associated with the largest eigenvalue, and x is the pattern coefficient or component score associated with the second largest eigenvalue. The value of sign(y) is 1 if y > 0 and –1 otherwise. The unit is radians. These calculations give positive arc length for positive y and negative arc length for negative y.

In summary, let *length* stand for the item length, *r*, in the item space; *arc-delta* indicate the arc length in the item space; and *arc-theta* stand for the arc length in the respondent space. Then, length estimates starting values for α_i, arc-delta estimates starting values for δ_i, and arc-theta estimates starting values for θ_j.

The simulation study

In order to investigate the effectiveness of proposed methods to estimate parameters of unidimensional unfolding model given in Equation 1, three test lengths were considered: 10, 20, and 40. Since generally in attitudinal surveys fewer items are used, small test size of 10 items was introduced in this study. Respondent's attitude, discrimination, and threshold parameters for simulated data were generated in a manner described in the dimensionality study. However, the delta parameters were generated from three different distributions to reflect realistic situations: item locations uniformly distributed over a wide range of the continuum (i.e., from -2 to +2); item locations more frequently distributed in the positive regions of the continuum (i.e., from -1 to +2); and item locations distributed only in the positive regions of the latent continuum (i.e., from 0 to +2). Individual responses for a given test length and delta distribution were generated based on Equation 1 in exactly the same manner explained for the dimensionality study.

For a given data set, three estimates arc-theta, arc-delta, and length were computed and for each parameter, correlations were computed between true (δ_i, α_i, and θ_j) and estimated

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[1] There are various formulas to compute the arc length that are mathematically equivalent.

values (arc-delta, length, arc-theta) for each condition of delta distribution. Moreover, for each parameter, regression analysis of true values of the parameter on estimated values was performed to see if the true scale of the parameter could be recovered. This process was replicated 100 times and the descriptive statistics of these estimates over 100 replications was computed. Results are reported in Tables 2, 3, and 4.

Results of the simulation study for d=1

Table 2 shows results for the location estimate arc-delta. Table values are the averages over 100 replications and are grouped according to the three distributions of item locations. As can be seen, the product moment correlation coefficients between true (δ) and estimated values (arc-delta) are extremely high for all test sizes and for all three distributions, indicating very strong association between the true and estimated values. However, as evidenced by the results of the regression analyses, the metric of estimated and the true values are not always the same. The slope and the intercept, although close to desirable values of one and zero for the wide distribution of deltas (-2 to 2), deviate from what is expected for the narrow distributions of delta. Furthermore, the standard deviations of intercepts are large in all cases.

Table 3 shows results for the respondent estimates (arc-theta). The correlations between θ and arc-theta are moderately high for all test sizes and in all three distributions. These correlations, as expected, are much larger for the widely spread distribution of location parameters (-2 to 2) than for other distributions of the location parameter. As in the case of arc-delta, the regression results of θ on arc-theta show that the estimates are not on the same metric as the original (θ) parameter. This is indicated by average slopes that are much less than unity. Additionally, the average intercept was greater than zero for narrow distribution of deltas.

Table 4 shows results for estimates of the discrimination parameter, α. As seen previously for δ and θ, the correlations between true and estimated values are moderate for all test sizes and all delta distributions. Again, as seen before, the metric of the estimated values is not the same as the true metric as evidenced by regression analyses results.

In summary, all three estimates, arc-theta, length, and arc-delta, have moderately high to extremely high correlations with their respective true parameters. However, they are not on the same metric as the original parameters. Several procedures were conducted in an attempt to

recover the original metric of the parameters, but none of these procedures produced acceptable results. Therefore, these efforts are not reported here.

Two-dimensional Case (d=2)

As described in the dimensionality study, when there are two well-defined unfoldable dimensions, and the test is constructed to represent the entire continuum, principal components analysis of such data yields four dominant linear components. By observing the plots of the pattern coefficients of the four components one can determine the pairs of components, associated with each dimension. In realistic situations neither dimensionality nor the items associated with each dimension is known. Hence after determining dimensionality underlying the data, it is necessary to correctly identify component pairs associated with each dimension, and items that form each dimension, before estimating item and respondent parameters.

In the two-dimensional unfolding case, there are four eigenvalues associated with the four linear dimensions. However, for unfolding data, two eigenvalues and the associated pattern coefficients define each unfoldable dimension. The first two largest eigenvalues generally correspond to the two unfoldable dimensions underlying data. Each of the remaining two eigenvalues is also associated with one of the two unfoldable dimensions. Similarly each of the first two sets of pattern coefficients is associated with one of the sets from the remaining components that defines each dimension, as shown through the simplex design of the pattern coefficients. An example may illustrate this point. Figures 2 to 7 show six plots of factor pairs associated with four linear dimensions of a 40-item two-dimensional test. From observing these plots it is clear that linear components 1 and 4 are associated with one unfolding dimension (Figure 4), and components 2 and 3 are associated with the other unfolding dimension (Figure 5). Plots in Figures 2, 3, 6, and 7 show mismatched components.

In terms of matching component pairs, since the largest components 1 and 2 are each associated with a different unfoldable dimension, they cannot be paired with each other. Similarly, components 3 and 4 cannot be paired together. Therefore possibilities for correct matching are only component pairs: 1 and 3, 1 and 4, 2 and 3, or 2 and 4. Two out of these four are correct matches. Hence for a given test, the task is to identify component-pairs and items associated with each dimension.

Several methods were tried to identify correct pairs of components and to correctly match items to pairs. The following algorithm seems most promising. At present this algorithm has been applied only to test items resembling a simple structure scenario. That is, each item is determined by only one attitude, either θ_1 or θ_2.

Algorithm 2: Algorithm to match items to correct component pairs based on "the weighted correspondent angle."

Here the idea is to first identify the pair of components that correspond to each simplex and then assign items to the appropriate simplex. An index called "the weighted correspondent angle" is computed for each item based on its length and the angle it makes with the nearest reference axis. The standard deviation of the weighted correspondent angles is generally largest for the pair of components that define a simplex pattern than for other component pairs. Once component pairs associated with simplexes are identified, then items are assigned to simplexes based on their length. The following steps are used to identify correct component pairs and match items to the correct simplex.

Step1. For each component pair, consider items only in the first and the fourth quadrants where the simplex pattern is situated.

Step2. Compute the angle between each included point and its closest reference axis. Multiply the length of the item vector with the angle it makes with the closest axis. Call this "the weighted correspondent angle."

Step3. Compute the standard deviation of weighted correspondent angles over all items.

Step4. Compare the standard deviations of correspondent angles for all logical component pairs: (1,3), (1,4), (2,3), and (2,4). The component pair associated with the largest standard deviation of correspondent angles provides the correct combination of components associated with one of the unfolding dimensions. The component pair for the second dimension is its compliment pair. For example, if the component pair (1,3) is associated with one dimension, then the component pair (2,4) is associated with the other dimension.

Step5. Assign items to one of the two selected component pairs defined (i.e., simplexes) in step 4: find the length of the item vector in each simplex; assign the item to the simplex where the length is largest.

The simulation study d=2

A simulation study was undertaken to investigate the effectiveness of the proposed algorithm to identify the component pairs defining each simplex and to correctly match items in each simplex. Individual item responses were generated in exactly the same manner as explained in the dimensionality study for the two-dimensional data. The test length was fixed to 40 items and respondent size was fixed to 1000 subjects. Three distributions of item locations were considered as before: item locations uniformly distributed over the entire range of the continuum from -2 to +2; item locations more frequently distributed in the positive regions from -1 to +2; and item locations distributed only in the positive regions of the latent continuum from 0 to +2. Algorithm 2 was applied for the data set in each distribution category, and results are reported in Table 5.

Results of the simulation study for d=2

Table 5 shows results for identifying correct component pairs (i.e., simplexes) and matching items to the correct simplex. The cell values are the percentages out of 100 replications, where all 40 items were correctly matched to dimensional groups after the two fan-like simplex patterns were identified. It can be seen that the percentages are very high in all cases, indicating the usefulness of Algorithm 2.

Observing replications where all items were not correctly classified into correct groups, it was found that in most cases the number of misclassified items were few. However, there were few replications (1 to 3 out of 100), especially when the deltas ranged between -1 and 2, where about half the number of items were misclassified. In realistic situations, one hopes that the content of the items and prior experience can aid in classifying such items into right simplex.

Summary and Discussion

Results reported in this study constitute ongoing research in dimensionality assessment, and development of start values for parameter estimation of test data, generated from attitudinal items that follow the generalized graded unfolding model. One- and two-dimensional simulated test data were analyzed in this study. Results so far are very encouraging. Principal components analysis provides a reliable estimate of the underlying dimensionality of unfolding data, namely, 2 linear dimensions are generated for every unfoldable dimension. In the case of unidimensional

unfolding data, parameter estimates arc-theta, arc-delta, and length, have high correlations with true parameters, indicating their usefulness as start values. It is, however, desirable to recover the true metric of these parameters, or at least a common metric for all parameter estimates.

For two-dimensional test data, several methods were attempted to identify the correct pair of components defining each unfoldable dimension; and to classify items correctly to these dimensions. Only Algorithm 2 has shown promise, and the results of Table 5 are very encouraging. This algorithm needs to be further validated on a broad variety of tests resembling realistic two-dimensional situations with varied items in each dimension and correlations between attitudinal dimensions.

This study is highly limited in many ways. These results cannot be generalized until a full scale simulation study is completed. Future studies will focus on also estimating the item threshold parameters in the generalized graded unfolding model in addition to the location and discrimination parameters. Another important focus of future studies would be to recover the metric of the original parameters (or at least a common metric among parameters) and computing initial estimates for two-dimensional tests.

With the recent emergence of a large class of IRT models for unfolding data, it is important to have methods to determine the fit of the models to data before applying these models. Given the sparse literature in dimensionality assessment for unfolding data, it is critical to perform a through investigation of dimensionality assessment in this area.

References

Andrich, D. (1988). The application of an unfolding model of the PIRT Type to the measurement of attitude. *Applied Psychological Measurement, 12*, 33-51.

Andrich, D., & Luo, G. (1993). A hyperbolic cosine latent trait model for unfolding dichotomous single-stimulas responses. *Applied Psychological Measurement, 17*, 253-276.

Buja, A., & Eyuboglu, N. (1992). Remarks on parallel analysis. Multivariate Behavioral Research, 27(4), 509-540.

Coombs, C. H. (1964). A Theory of Data. New York: Wiley.

Davison, M. (1977). On a metric, unidimensional unfolding model for attitudinal and developmental data. *Psychometrika, 42*, 523-548.

Hoijtink, H. (1990). A latent trait model for dichotomous choice data. *Psychometrika, 55*, 641-656.

Hoijtink, H. (1991). The measurement of latent traits by proximity items. *Applied Psychological Measurement, 15*, 153-169.

Luo, G. (2001). A class of probabilistic unfolding models for polytomous responses. Journal of Mathematical Psychology, 45, 224-248.

Mauraun, M. D., & Rossi, N. T. (2001). The extra-factor phenomenon revisited: unidimensional unfolding as quadratic factor analysis. *Applied Psychological Measurement, 25*, 77-87.

Roberts, J. S. & Laughlin, J. E. (1996). A unidimensional item response model for unfolding responses from a graded disagree-agree response scale. *Applied Psychological Measurement, 20*, 231-255.

Roberts, J. S., Donoghue, J. R., & Laughlin, J. E. (2000). A general item response theory model for unfolding unidimensional polytomous responses. *Applied Psychological Measurement, 24*, 3-32.

Ross, J. & Cliff, N. (1964). A generalization of the interpoint distance model. *Psychometrika, 29*, 167-176.

Thurstone, L. L. (1927). A law of comparative judgement. *Psychological Review, 34*, 278-286.

Thurstone, L. L. (1928). Attitudes can be measured. *American Journal of Sociology, 33*, 529-554.

van Schuur, W. H. (1984). Structure in political beliefs: A new model for stochastic unfolding with application to European party activists. Amsterdam: CT Press.

van Schuur, W. H., & Kiers, H. A. L. (1994). Why factor analysis is often the incorrect model for analyzing bipolar concepts, and what model can be used instead. *Applied Psychological Measurement, 18*, 97-110.

Table 2: Summary of Results for arc-delta*										
		-2 < delta < 2			-1 < delta < 2			0 < delta < 2		
test size		10	20	40	10	20	40	10	20	40
r(delta,arc-delta)		0.98	0.99	0.99	0.98	0.99	0.99	0.95	0.97	0.96
R-squared		0.97	0.98	0.98	0.96	0.96	0.97	0.91	0.93	0.93
Regression of delta on arc-delta	intercept	0.05	0.00	0.02	0.00	0.00	0.00	0.04	-0.38	-0.50
	slope	0.95	0.94	0.95	0.90	0.89	0.90	0.95	1.00	1.00
	sd (int)	0.31	0.21	0.15	0.48	0.37	0.34	0.98	0.56	0.13
	sd(slope)	0.10	0.04	0.03	0.10	0.08	0.03	0.10	0.06	0.05

*Except where identified by "sd," entries in all tables are averages over 100 replications. Entries marked "sd" are standard deviations over 100 replications.

Table 3: Summary of Results for arc-theta*										
		-2 < delta < 2			-1 < delta < 2			0 < delta < 2		
test size		10	20	40	10	20	40	10	20	40
r(theta,arc-theta)		0.84	0.92	0.95	0.73	0.85	0.88	0.71	0.79	0.82
R-squared		0.72	0.85	0.90	0.60	0.75	0.81	0.53	0.63	0.68
Regression of theta on arc-theta	intercept	0.01	0.00	0.01	0.00	0.01	0.00	0.10	0.13	-0.13
	slope	0.55	0.62	0.65	0.48	0.57	0.59	0.46	0.52	0.55
	sd (int)	0.11	0.11	0.09	0.11	0.13	0.17	0.05	0.04	0.04
	sd(slope)	0.11	0.04	0.02	0.16	0.12	0.11	0.11	0.08	0.05

Table 4: Summary of Results for length										
		-2 < delta < 2			-1 < delta < 2			0 < delta < 2		
test size		10	20	40	10	20	40	10	20	40
r(delta,length)		0.83	0.87	0.85	0.84	0.84	0.84	0.79	0.84	0.85
R-squared		0.70	0.75	0.73	0.72	0.71	0.72	0.64	0.71	0.72
Regression of alpha on length	intercept	-3.29	[illegible]3	-1.87	-3.22	-2.09	-1.67	-3.09	2.22	-1.85
	slope	5.69	4.69	4.16	5.67	4.42	3.94	5.33	4.52	4.12
	sd (int)	1.22	0.47	0.36	1.12	0.55	0.32	1.53	0.57	0.33
	sd(slope)	1.50	0.61	0.47	1.38	0.71	0.42	1.90	0.71	0.42

Table 5: Percentage of Correct Classification of Items to Dimensions			
	-2 < delta < 2	-1 < delta < 2	0 < delta < 2
test size	40	40	40
Algorithm3	94	88	91

Table1: Results of Dimensionality Analysis of Unfolding Data									
					Number of factors**				
dim	#items	#th1-#th2-#mix	s1, s2*	Corr between th1 and th2	PCEX	PCOB	FAEX	FAOB	eigenvalues(PCOB)***
1	80	-	-	-	2	2	4	4	31.10, 14.61, 1.06, 1.04, .77, .76
1	20			-	2	2	3	5	7.94, 4.34, .71, .65, .57, .53
2	80	40-40-0	-	0	4	4	7	6	17.38, 14.94, 6.68, 6.49, .95, .86
2	40	20-20-0	-	0	4	4	5	5	8.75, 8.26, 3.58, 3.21, .80, .74
2	20	10-10-0	-	0	4	4	5	13	5.40, 4.7, 2.07, 1.26, .74, .70
2	80	0-0-80	0.5, 0.5	0	4	4	5	4	20.32, 9.8, 3.48, 2.60, .98, .96
2	80	0-0-80	0.5, 0.5	0.75	4	4	5	4	23.38, 14.10, 1.81, 1.42, .97, .94
2	80	35-35-10	0.5, 0.5	0	4	4	8	7	15.62, 13.40, 8.10, 6.06, 1.25, .96
2	80	40-40-0	-	0.3	4	4	7	6	19.31, 11.39, 8.63, 6.39, .93, .85
2	80	0-0-80	.75, .25	0	4	4	6	5	23.82, 10.64, 2.38, 1.66, 1.00, .96
2	80	0-0-80	.75, .25	0.3	4	4	6	5	
2	80	0-0-80	.75, .25	0.5	3	3	6	5	28.08, 14.65, 1.17,1.11,.95, .91
	* s1 and a2 are the weights given to the two dimesnions. s1=0.75 and s2=0.25, denotes that theta1 was given higher weight than theta2.								
	** the number of factors are averaged over 5 replications								
	*** eigen values for one replication - PC of observed data								

Figure 1. Plot of pat1*pat2$lab. Symbol used is '*'.

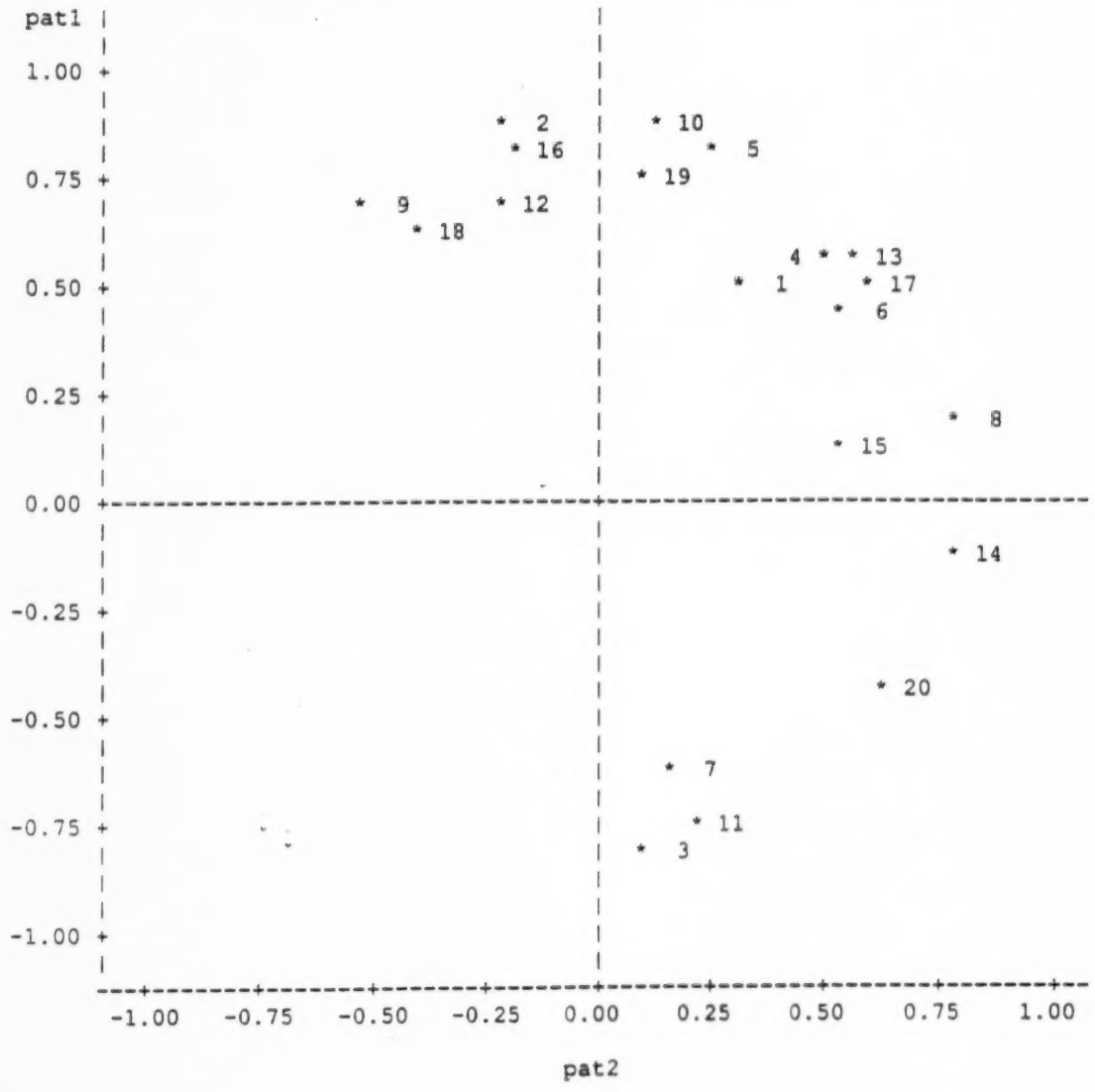
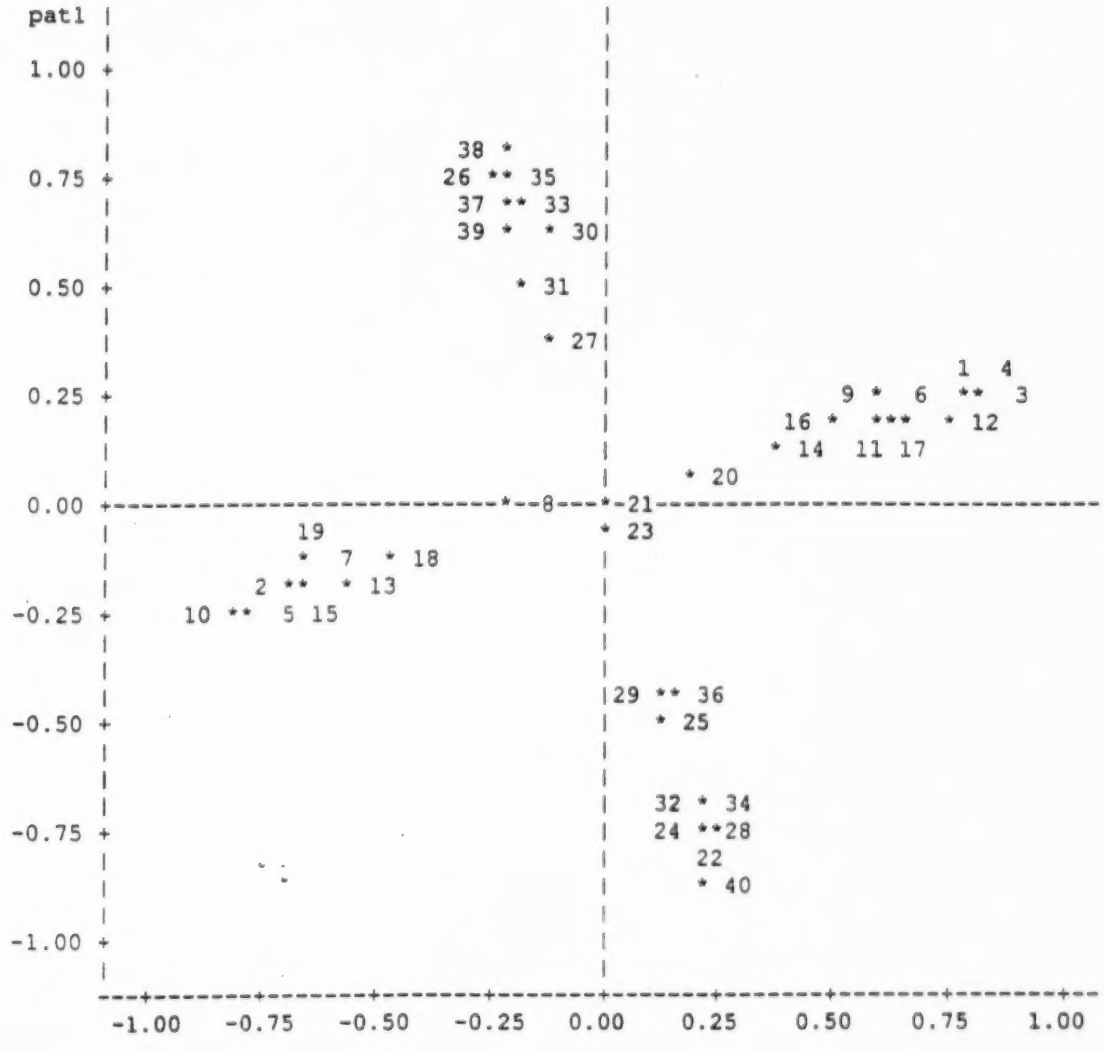
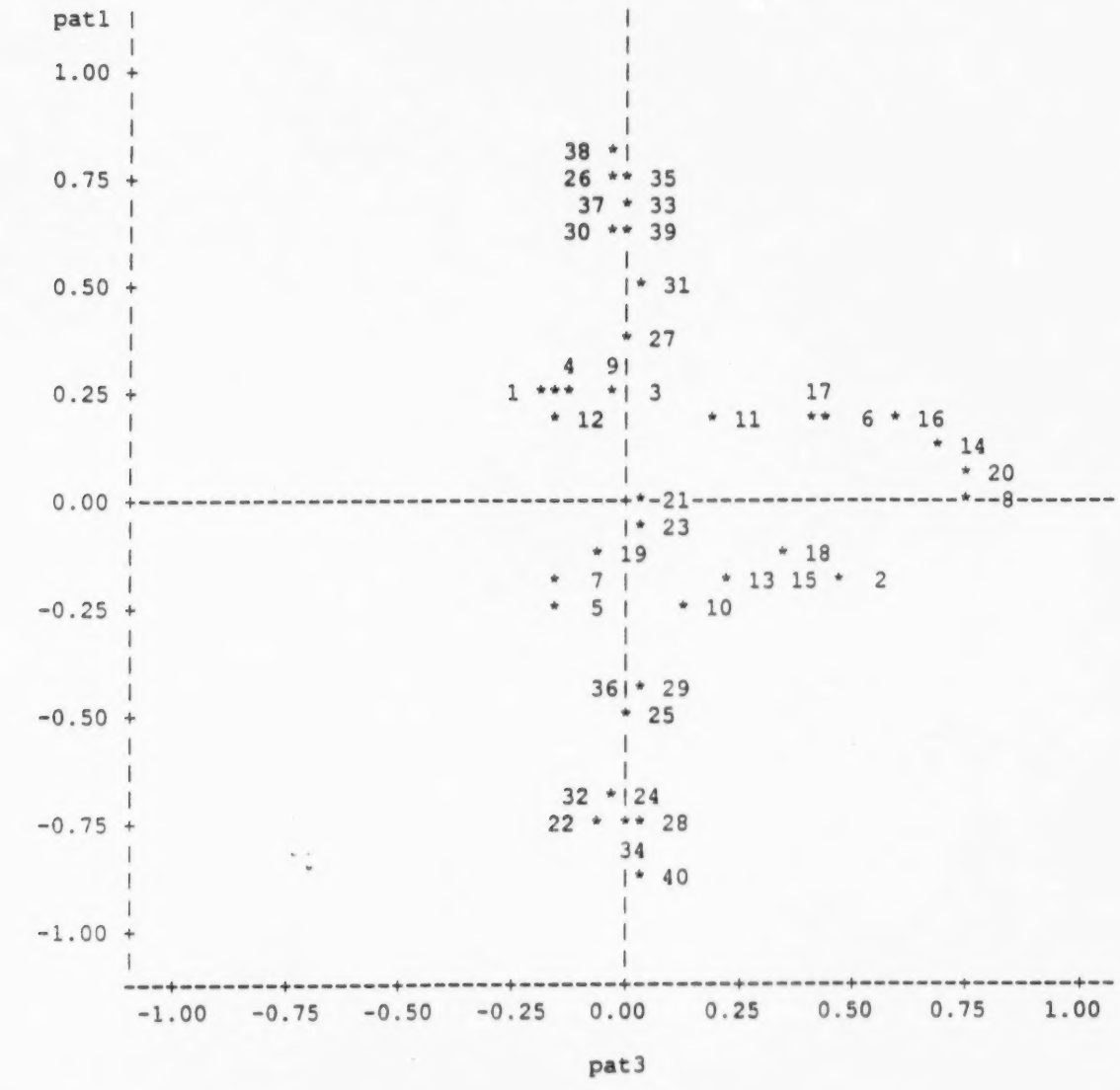


Figure 2. Plot of pat1*pat2$lab. Symbol used is '*'.



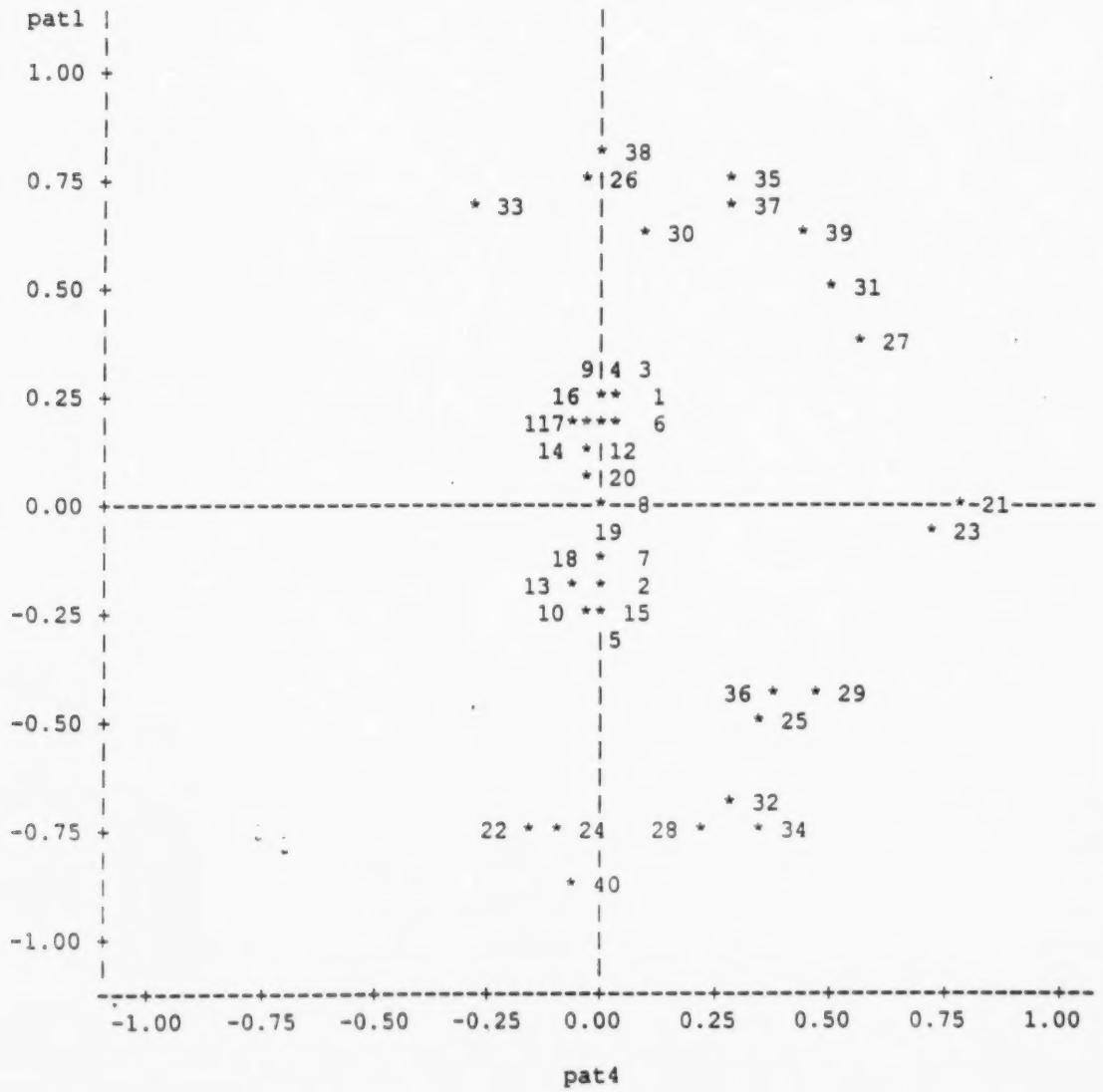
NOTE: 4 obs hidden.

Figure 3. Plot of pat1*pat3$lab. Symbol used is '*'.



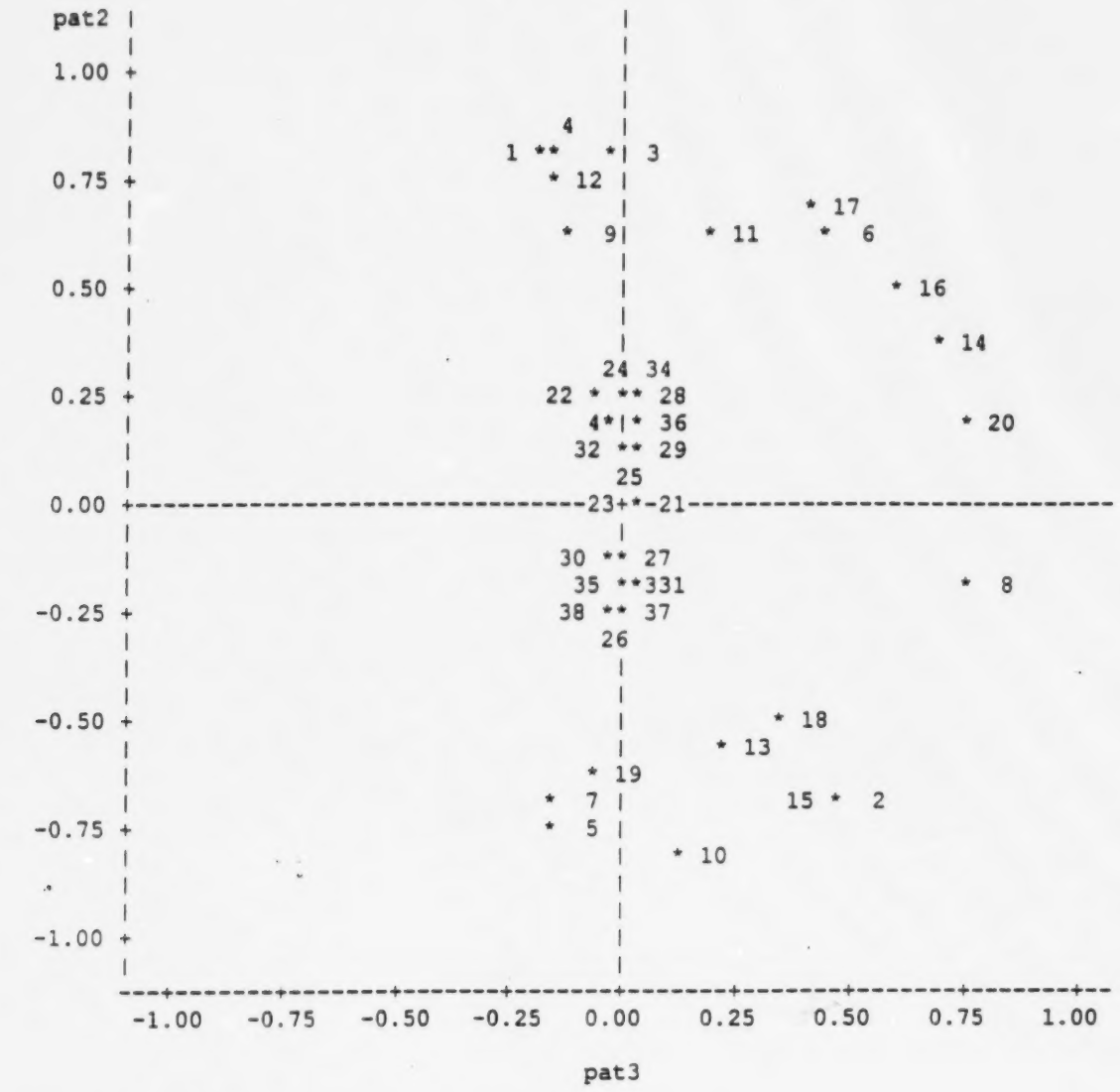
NOTE: 4 obs hidden.

Figure 4. Plot of pat1*pat4$lab. Symbol used is '*'.



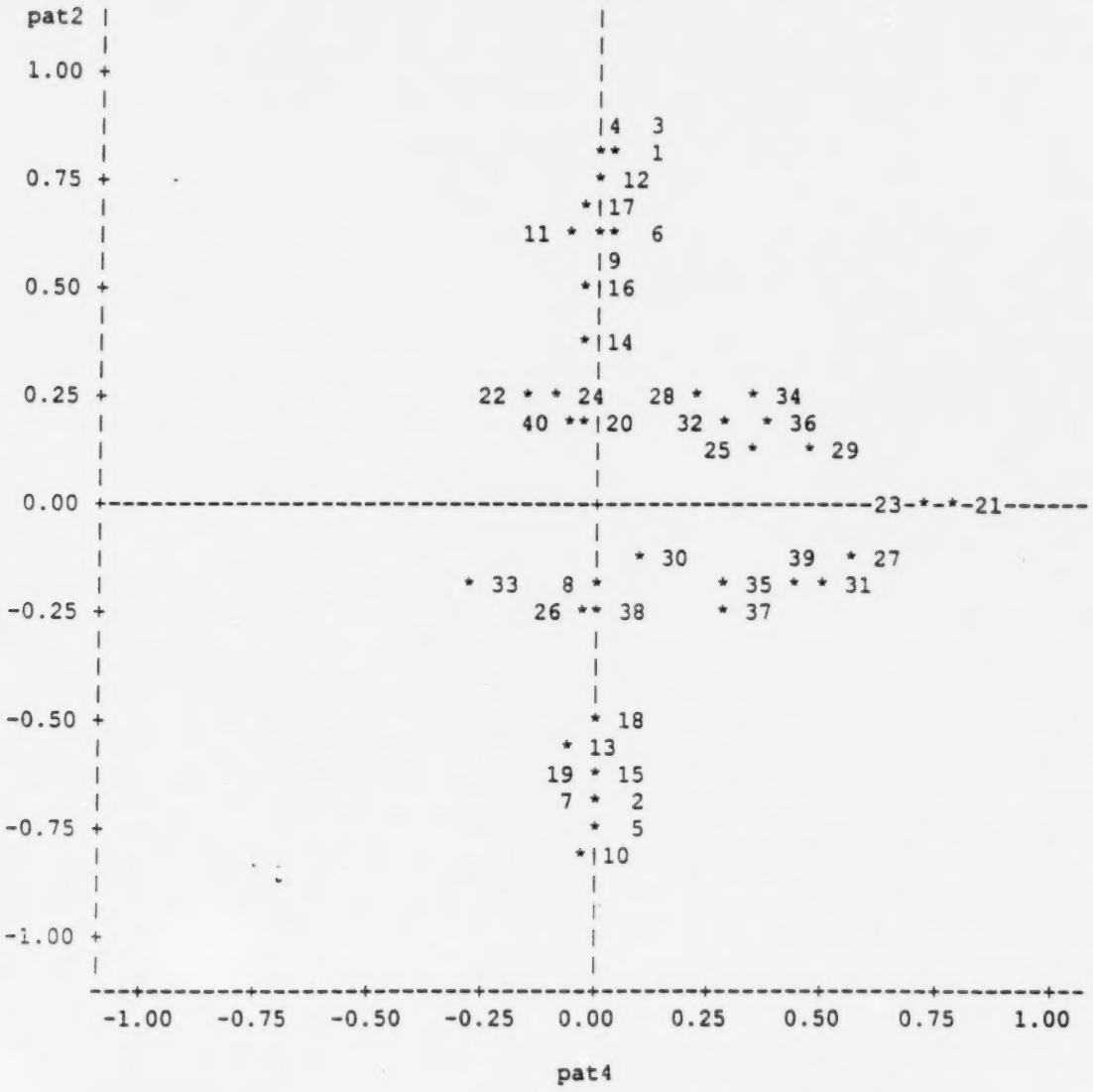
NOTE: 6 obs hidden.

Figure 5. Plot of pat2*pat3$lab. Symbol used is '**'.



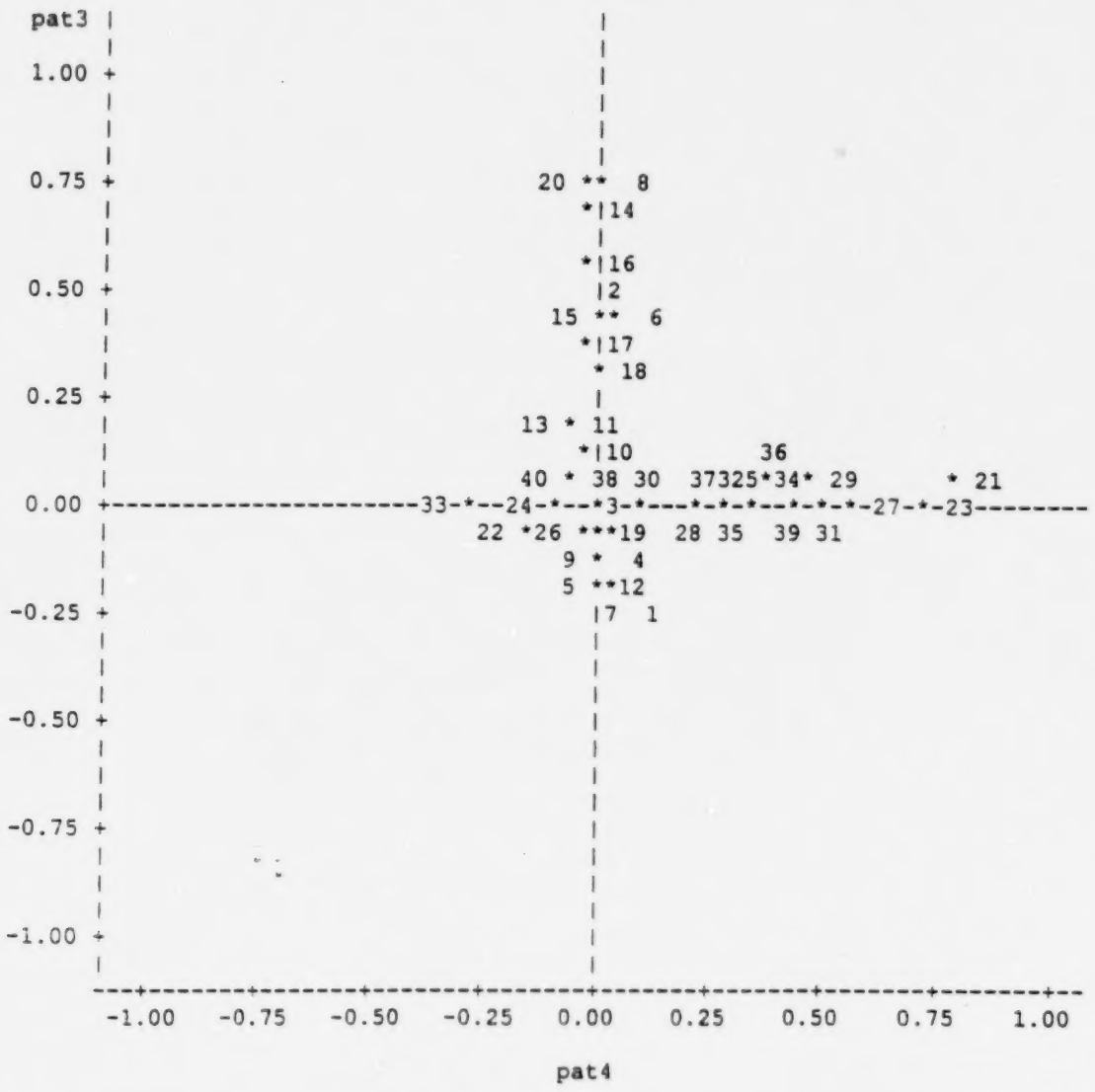
NOTE: 7 obs hidden. 2 label characters hidden.

Figure 6. Plot of pat2*pat4$lab. Symbol used is '*'.



NOTE: 3 obs hidden.

Figure 7. Plot of pat3*pat4$lab. Symbol used is '*'.



NOTE: 8 obs hidden.